Atmospheric Dynamics
11:670:324

Class Time: Tuesdays and Fridays 9:15-10:35

Instructors: Dr. Anthony J. Broccoli (ENR 229)
broccoli@envisci.rutgers.edu
848-932-5749
Dr. Benjamin Lintner (ENR 250)
lintner@envisci.rutgers.edu
848-932-5731

TAs: Jenny Kafka
Arielle Alpert

Website: Atmospheric Dynamics Sp14 on Sakai (sakai.rutgers.edu)


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**Grading**

- Quiz (mathematical methods, including vector analysis): 5%
- Homework problems: 15%
- GEMPAK exercises: 15%
- First hourly exam: 20%
- Second hourly exam: 20%
- Final exam: 25%
What Are Dynamics?

- **Definition**: The study of atmospheric and oceanic motions, with emphasis on the physical laws that govern such motions.

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Course Objectives

- To lay a mathematical and theoretical foundation to be used in later applications.
- To apply the laws governing fluid motion (laws of hydrodynamics and thermodynamics) to the atmosphere in order to understand and predict its behavior.
- To share with you the excitement of how mathematics can be used to describe what occurs in the real world.
NYC: December 25-27, 1947

- U.S. Weather Bureau forecast, Dec. 25, 9:30 PM
  "Tonight: cloudy with some snow possible toward morning. Friday—Cloudy with occasional snow ending during the afternoon followed by partial clearing."

- U.S. Weather Bureau special bulletin, Dec. 25, 9:50 PM
  "Cloudy with snow towards morning ending during Friday afternoon, lowest temperature near 25. This is a borderline situation and the amount and time of precipitation depends on the movement of storm centered at 7:30 this evening off the Carolina coast. There is 1 chance in 4 that snow may be heavy and last most of the day, and 1 chance in 4 that no or little snow may fall."

- Dec. 26, 7:00 AM: Nearly 2" of snow has fallen.
- Dec. 26, 1:00 PM: Snowfall reaches 12" with snow still falling.
- Dec. 26, 7:00 PM: Snowfall reaches 24", stranding many commuters.
- Dec. 27, early morning: Snow ends; 26.4" measured at Central Park.

Source: Kocin and Uccellini, Northeast Snowstorms, American Meteorological Society
Learning Goals

• Develop a conceptual understanding of atmospheric dynamical processes.
• Master the foundational mathematical and physical principles of atmospheric dynamics.
• Apply the conceptual understanding and mathematical and physical principles to solve problems.
• Use specialized software to analyze real-time and historical meteorological data.

Basic Laws

• Conservation of mass (continuity equation)
• Conservation of energy (1st law of thermodynamics)
• Newton’s 1st Law (no resultant force → no change in motion)
• Newton’s 2nd Law (rate of change of motion of a body is proportional to resultant force acting on it)
• Conservation of angular momentum
• Newton’s Law of Gravitation
• Ideal Gas Law (equation of state)
Coordinate Systems

- To describe the location in space of a point in a fluid, a coordinate system is used.
- A commonly used coordinate system is the rectangular, or $x,y,z$ system (also known as Cartesian).

- Rectangular coordinates are often used to describe motions of the atmosphere or ocean, even though the earth is a sphere.
- In so doing, one assumes that the $x$-$y$ plane is tangent to the surface of the spherical earth.
- General convention for use of rectangular coordinates:
  - $x$ is a measure of distance from some origin and increases toward the east.
  - $y$ is a measure of distance from some origin and increases toward the north.
  - $z$ is zero at surface of earth and increases upward.
Fundamental Mathematical Concepts and Operations

- Fundamental state variables such as wind speed, temperature and pressure are functions of (i.e., depend upon) the independent variables \((x, y, z, t)\).
- For example, atmospheric pressure can be expressed as a function of space and time:

\[
P = P(x, y, z, t)
\]

Derivatives

Assume \(\Delta x\) represents a small distance in the \(x\) direction.

The quotient \(\frac{\Delta f}{\Delta x}\) represents the slope.

The derivative of a function \(f(x)\) is defined as

\[
\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}
\]

In the limit (as \(\Delta x\) goes to 0), this becomes the slope at a point and this is the derivative \((df/dx)\), or the gradient or rate of change.
Partial Derivatives

With standard derivatives, our function varied in one dimension. However, some variables such as temperature vary not only in time, but also in space: \( T(x, y, z, t) \)

The partial derivative of \( T \) with respect to \( x \) will tell us how fast \( T \) changes as we move in the \( x \) direction and is defined as follows:

\[
\frac{\partial T}{\partial x} = \lim_{\Delta x \to 0} \frac{T(x + \Delta x, y, z, t) - T(x, y, z, t)}{\Delta x}
\]

Similarly,

\[
\frac{\partial T}{\partial y} = \lim_{\Delta y \to 0} \frac{T(x, y + \Delta y, z, t) - T(x, y, z, t)}{\Delta y}
\]

Chain Rule Of Differentiation

Assume:

\[
f = f(u, v)
\]

\[
u = u(x, y)
\]

\[
v = v(x, y)
\]

Then:

\[
\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u} \frac{\partial u}{\partial x} + \frac{\partial f}{\partial v} \frac{\partial v}{\partial x}
\]

\[
\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u} \frac{\partial u}{\partial y} + \frac{\partial f}{\partial v} \frac{\partial v}{\partial y}
\]
More Identities

\[ \frac{\partial (uv)}{\partial x} = u \frac{\partial v}{\partial x} + v \frac{\partial u}{\partial x} \]

Order of partial differentiation does not matter.

\[ \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial x} \right) = \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial y} \right) \]

\[ \frac{\partial (\ln f)}{\partial x} = \frac{1}{f} \frac{\partial f}{\partial x} \]

Expansion of Total Derivative

If \( f = f(x, y, z, t) \) then

\[ \frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} + \frac{\partial f}{\partial z} \frac{dz}{dt} \]

But \( u \equiv \frac{dx}{dt}, \quad v \equiv \frac{dy}{dt}, \quad w \equiv \frac{dz}{dt} \)

\( u = \) west-east component of fluid velocity
\( v = \) south-north component of fluid velocity
\( w = \) vertical component of fluid velocity
Euler’s relation (expansion of total derivative):

\[
\frac{df}{dt} = \frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} + v \frac{\partial f}{\partial y} + w \frac{\partial f}{\partial z}
\]

Term A: Local rate of change of \(f\) at a fixed location

Term B: Total rate of change of \(f\) following the fluid motion

Term C: Advection of \(f\) in \(x\) direction by the \(x\)-component flow

Term D: Advection of \(f\) in \(y\) direction by the \(y\)-component flow

Term E: Advection of \(f\) in \(z\) direction by the \(z\)-component flow

Total Derivative vs. Local Derivative

**Total derivative** is the temporal rate of change following the fluid motion. Example: A thermometer measuring changes as a balloon floats through the atmosphere.

**Local derivative** is the temporal rate of change at a fixed point. Example: An observer measures changes in temperature at a weather station.
Advection Terms

Assume that thin lines are contours of a scalar quantity \( f \) and thick arrows indicate the fluid motion. We wish to evaluate the advection term \( -u \frac{\partial f}{\partial x} \).

At point A: \( u > 0, \frac{\partial f}{\partial x} > 0 \rightarrow -u \frac{\partial f}{\partial x} < 0 \) ➠ Transport from low to high: “negative advection of \( f \)”

At point B: \( u = 0, \frac{\partial f}{\partial x} > 0 \rightarrow -u \frac{\partial f}{\partial x} = 0 \) ➠ “neutral advection of \( f \)”

At point C: \( u < 0, \frac{\partial f}{\partial x} > 0 \rightarrow -u \frac{\partial f}{\partial x} > 0 \) ➠ Transport from high to low: “positive advection of \( f \)”

Taylor Series

A function \( f(x) \) can be computed by Taylor expansion given the values of the function and its derivatives at a point \( x_0 \):

\[
f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3 + \ldots \\
f(x) = f(x_0) + \sum_{n=1}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n
\]

A truncated Taylor series can be used to approximate \( f(x) \).