Lecture 2 Topics

• Part I: Radiative-convective equilibrium (RCE); convective quasi-equilibrium (QCE)
• Part II: Convective criticality and column water vapor precipitation statistics (Kathleen)
Vertical temperature profile

What determines this profile?
Thermal emission and the Planck function

The Planck function determines the amount of energy per unit time per area per wavelength radiated by an object at temperature $T$:

$$R(\lambda, T) = \frac{2hc^2}{\lambda^5 \left(e^{hc/\lambda k_B T} - 1\right)}$$

$h$ [Planck’s constant] = $6.625 \times 10^{-34}$ J s$^{-1}$

$c$ [speed of light] = $3.0 \times 10^8$ m s$^{-1}$

$k_B$ [Boltzmann’s constant] = $1.38 \times 10^{-23}$ J K$^{-1}$

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Simple radiative balance for a planet (with no atmosphere)

Integrating the Planck function over all wavelengths and multiplying by the surface area of a planet of radius R:

\[ F = 4\pi R^2 \sigma T^4 \]

\[ \sigma = 5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4} \]

For a planetary albedo \( a \) with a solar flux density \( S_0 \), the absorbed incoming solar radiation is:

\[ S_{\text{abs}} = S_0 (1 - a) \pi R^2 \]

At thermal equilibrium:

\[ F = S_{\text{abs}} \Rightarrow T_e = \left[ \frac{S_0 (1 - a)}{(4\sigma)} \right]^{1/4} \]

For parameters appropriate to Earth, this yields an effective emission (surface) temperature of 255K (-18°C), compared to an observed surface temperature of 288K (15°C).
Mean tropospheric composition

<table>
<thead>
<tr>
<th>Constituent</th>
<th>% by volume of dry air</th>
</tr>
</thead>
<tbody>
<tr>
<td>N₂</td>
<td>78.08</td>
</tr>
<tr>
<td>O₂</td>
<td>20.95</td>
</tr>
<tr>
<td>CO₂</td>
<td>0.033 ↑</td>
</tr>
<tr>
<td>Ar</td>
<td>0.934</td>
</tr>
<tr>
<td>Ne</td>
<td>1.82 x 10⁻³</td>
</tr>
<tr>
<td>He</td>
<td>5.24 x 10⁻⁴</td>
</tr>
<tr>
<td>CH₄</td>
<td>2.0 x 10⁻⁴ ↑</td>
</tr>
<tr>
<td>Kr</td>
<td>1.14 x 10⁻⁴</td>
</tr>
<tr>
<td>N₂O</td>
<td>5.0 x 10⁻⁵ ↑</td>
</tr>
<tr>
<td>H₂</td>
<td>5.0 x 10⁻⁵</td>
</tr>
<tr>
<td>Xe</td>
<td>8.7 x 10⁻⁶</td>
</tr>
<tr>
<td>O₃</td>
<td>4.0 x 10⁻⁶</td>
</tr>
<tr>
<td>H₂O</td>
<td>0.0-4.0 [0.8]</td>
</tr>
</tbody>
</table>

F.W. Taylor, *Elementary Climate Physics*
Atmospheric absorption

In the visible part of the electromagnetic spectrum, Earth’s atmosphere is effectively transparent ("solar window"). At shorter and longer wavelengths, on the other hand, the atmosphere is typically strongly absorbing (except for the “atmospheric window”).

Turco [2002]
Radiative balance for a single layer atmosphere

\[ \sigma T_s^4 = \sigma T_a^4 + \sigma T_e^4 \implies T_s = 2^{1/4} T_e \]  

Surface temperature (~303K) is now too large…
Pure radiative equilibrium yields an unstable profile in the troposphere, although it is quite reasonable in the stratosphere.
The vertical transport of heat via convection counteracts the radiatively-induced instability in the troposphere.

Emanuel [2005]
Simulations are performed with a 1D single column model with a convection parameterization and albedo tuned to give a reasonable sounding at the equator. Note that mean surface temperatures are below 0 poleward of 35 degrees.

RCE predicts: height of the tropopause, ~constant relative humidity with warming, precipitation equal evaporation.
Convection as events

This profile contains a large amount of CAPE, suggesting the possibility of very intense convection. For convection to be realized, the CIN needs to be overcome.

We see this situation over summertime midlatitude continents.

But in the tropics (especially over the oceans), the atmosphere is frequently quite close to moist adiabatic and neutral stability.
Convection as a statistical equilibrium: convective quasi-equilibrium (CQE)

Because the tropical free troposphere is frequently observed to be close to moist adiabatic, it is posited that the effect of convection is maintaining a moist adiabatic (and neutrally stable) tropical atmosphere troposphere.

The underlying notion of the statistical equilibrium view of convection is that the small-scale cumulus convection is comparable to the large-scale processes with which it interacts.

*Arakawa & Schubert* [1974] first applied this notion as a basis for convective closure. Convective closure refers to the necessity in coarse resolution models of representing the inherently subgrid-scale convective process in terms of the resolved grid-scale [as we will discuss further in Lecture 4].
The 2010 Vilhelm Bjerknes Medal is awarded to Akio Arakawa in recognition of his pioneering and fundamental contributions to physically based discretisation techniques in atmosphere and ocean models and to representations of convective clouds in atmosphere models, and for his continuing work on bridging the gap between the resolved and unresolved scales in atmospheric general circulation modelling.”
Cloud population
Cloud subensembles and work functions

Consider a quantity $\lambda$ that denotes a subensemble of convective clouds, characterized by a common attribute such as the altitude of the cloud tops. If $M_{\lambda}$ and $B_{\lambda}$ represent, respectively, the vertical mass flux and buoyancy of the $\lambda$ subensemble, the time rate of change of energy consumed (conversion of potential energy to kinetic) by the $\lambda$ subensemble is:

$$
\left(-\frac{dA_{\lambda}}{dt}\right)_C = \int_{z_{b,\lambda}}^{z_{t,\lambda}} M_{\lambda}B_{\lambda} dz
$$

Here, the limits of integration are the cloud base and top, $A_{\lambda}$ is the cloud work function, and the subscript $c$ denotes a convective- (cloud-) scale process.

On the other hand, the rate at which energy is generated, either by the large-scale or through interactions among subensembles, is:

$$
\left(\frac{dA_{\lambda}}{dt}\right)_{LS} = F_{\lambda} - (1 - \delta_{\lambda\lambda'}) \int_{z_{b,\lambda'}}^{z_{t,\lambda'}} M_{\lambda'}J_{\lambda\lambda'} dz
$$

Here, $F_{\lambda}$ is the explicit LS rate of energy production and $J_{\lambda\lambda'}$ accounts for interactions between the $\lambda$ and $\lambda'$ subensembles.  

*Arakawa & Schubert [1974]*
Interaction terms

Influence of vertical mass flux in $\lambda'$ subensemble clouds on $\lambda$ clouds (top) and $\lambda$ clouds on $\lambda'$ clouds (bottom).

Influence of cloud-top detrainment of $\lambda'$ subensemble clouds on $\lambda$ clouds. Note for the interaction as shown, the effect of $\lambda$ clouds on $\lambda'$ clouds is zero, since the detrainment in the former occurs above the latter.

Arakawa & Schubert [1974]
Application of QCE

CQE implies that the total time rate of change of the cloud work function is zero, implying the rate of LS energy production is equal to its consumption at the convective scale, i.e.,

\[
\frac{dA_{\lambda}}{dt} = \left(\frac{dA_{\lambda}}{dt}\right)_C + \left(\frac{dA_{\lambda}}{dt}\right)_{LS} = 0
\]

\[
\Rightarrow \left(\frac{dA_{\lambda}}{dt}\right)_{LS} = -\left(\frac{dA_{\lambda}}{dt}\right)_C
\]

That is:

\[
F_{\lambda} = \int_{z_{b\lambda}}^{z_{i\lambda}} M_{\lambda'} K_{\lambda\lambda'} dz
\]

where the integration kernel \( K_{\lambda\lambda'} \) is defined as:

\[
K_{\lambda\lambda'} = (1 - \delta_{\lambda\lambda'}) J_{\lambda\lambda'} + \delta_{\lambda\lambda'} B_{\lambda}
\]

Given \( F_{\lambda} \) and a cloud model that determines \( B_{\lambda}, M_{\lambda} \) can be determined: this is the basis of Arakawa & Schubert [1974] type convection schemes used in models. However, this determination is nontrivial, as it depends on the features of the cloud models and how they treat cloud microphysics.
Estimated tendencies of $A$ from observations in the Marshall Islands [Yanai 1973] suggest little time variation of work function, consistent with the CQE hypothesis.

CQE is often framed with CAPE replacing the work function. For tropical conditions, net surface flux and column radiative cooling generate ~4000 J kg$^{-1}$ day$^{-1}$, while CAPE values in the tropical atmosphere are typically below 1000 J kg$^{-1}$.

*Arakawa & Schubert [1974]*
Criticisms or limits of CQE

**Causality:** The formulation of CQE is often understood in terms of the large-scale forcing convection (thermodynamic analogy to a system responding to its environment), but what about the opposite direction, namely convection forcing the large-scale?

**Lack of (evidence for) scale separation:** Temporally, the convective timescale should be short compared to timescale of evolution of the large-scale. Also, the statistical view suggests that convective elements should be sufficiently small compared to the domain size, so that a large number can exist within the domain.

**Applicability to land/midlatitudes:** The presence of a strong boundary layer cycle (with convection happening preferentially in late afternoon) suggests a trigger is required to realize conditional instability.

**Transition to strong convection:** The indication of a clear onset threshold (in moisture, as a proxy for instability) challenges the equilibrium view.
Interpretations and alternatives to CQE*

*Following here the recent review by Yano & Plant [2012]. Schematic slightly embellished.
Relating ABL θ* and cumulus convection

Consider the CAPE defined as: \( CAPE = \int_{z_{LFC}}^{z_{EL}} g \rho^{-1} (\rho' - \rho) dz \). Using the definition of specific volume and transforming the integral to one over pressure [through hydrostatic equilibrium] gives:

\[ CAPE = \int_{p_{EL}}^{p_{LFC}} (\alpha - \alpha') dp \]

Under CQE, we have:

\[
\frac{\partial CAPE}{\partial t} = \int_{p_{EL}}^{p_{LFC}} (\frac{\partial \alpha}{\partial t} - \frac{\partial \alpha'}{\partial t}) dp \approx 0
\]

We can relate constant-pressure perturbations in parcel specific volume to perturbations in entropy via Maxwell’s relations. Thus,

\[
\left( \frac{\partial \alpha}{\partial t} \right)_p = \lim_{t \to 0} \frac{(\delta \alpha)_p}{\delta t} = \lim_{t \to 0} \left( \frac{\partial \alpha}{\partial s^*} \right)_p \frac{\delta s^*}{\delta t} = \left( \frac{\partial T}{\partial p} \right)_{s^*} \frac{\partial s^*}{\partial t} \quad s^* = c_p \ln \left( \frac{T}{T_0} \right) - R_d \ln \left( \frac{p}{p_0} \right) + \frac{L_e q_s}{T}
\]

On the other hand, from the hydrostatic relationship, expressed in terms of geopotential:

\[
\left( \frac{\partial \alpha'}{\partial t} \right)_p = \frac{\partial}{\partial t} \left( -\frac{\partial \Phi}{\partial p} \right)
\]

Emanuel et al. [1994]
ABL $\theta_e$ and cumulus convection (cont’d)

Thus:

$$\frac{\partial \text{CAPE}}{\partial t} = \int_{p_{EL}}^{p_{LFC}} \left[ \left( \frac{\partial T}{\partial p} \right) s^* \frac{\partial s^*}{\partial t} + \frac{\partial}{\partial t} \left( \frac{\partial \Phi}{\partial p} \right) \right] dp \approx 0$$

And:

$$\frac{\partial (c_p \ln \theta_e)}{\partial t} b (T_{LFC} - T_{EL}) + \frac{\partial}{\partial t} (\Phi_{LFC} - \Phi_{EL}) \approx 0$$

We have assumed saturation entropy [or $c_p \ln \theta_e$] is is conserved following ascent within the convecting cloud and can therefore be described by its ABL value.

The above relationship, namely that temporal fluctuations in the “thickness” of the CQE convecting layer are proportional to ABL fluctuations in moist entropy/equivalent potential temperature, is a fundamental result. It implies that a positive perturbation in ABL $\theta_e$ produces a positive perturbation in thickness above the ABL (note: $\Phi_{EL} - \Phi_{LFC} > 0$). Thus, to a large extent in the Tropics, predicting the response of ABL $\theta_e$ to disturbances determines the impact of such disturbances on convection.

Emanuel et al. [1994]
ABL $\theta_e$ and cumulus convection (cont’d)

The relationship derived on the previous slide allows us to cast the question of controls on convection in terms of controls on $\theta_e$:

$$\left(\ln\theta_e\right)_{b\,eq} \approx \frac{1}{w_0 + w_e + M_d} \times \left[ w_0 \ln(\theta_e) + w_e \ln(\theta_e) + M_d \ln(\theta_e) + \frac{z_b \dot{\mathcal{Q}}_{rad}}{c_p T} \right]$$

**Equilibrium subcloud $\theta_e$ increases with:**

1. Increasing SST [i.e., higher $\theta_{es}$]
2. Increasing surface windspeed [i.e., higher $w_0$]
3. Increasing above subcloud layer equivalent potential temperature [i.e., higher $\theta_{ee}$]
4. Increasing downdraft equivalent potential temperature [i.e., higher $\theta_{ed}$]

*Emanuel et al. [1994]*