Saturation mixing ratio

Recalling the definition of mixing ratio, \( w = \frac{\epsilon e}{p - e} \approx \frac{\epsilon e}{p} \),

we can define the **saturation mixing ratio** as: \( w_s = \frac{\epsilon e_s(T)}{p - e_s(T)} \approx \frac{\epsilon e_s(T)}{p} \)

There is a unique value of \( w_s \) for a given \( T \) and \( p \). We can therefore define lines of constant mixing ratio or vapor lines on a skew-T diagram.

Also, recall that \( \epsilon = 0.622 \), so for \( p = 622 \text{ mb} \), \( w_s(T, 622 \text{ mb}) = 10^{-3} e_s(T) \). Thus, \( w_s \) in units of g/kg corresponds to \( e_s(T) \) in units of mb.
Dewpoint on a skew-T

Consider a parcel of air with temperature $T$ and dewpoint temperature $T_d$ at pressure $p$. On a skew-T, the vapor line passing through the parcel’s dewpoint gives the parcel’s actual mixing ratio, because by definition of $T$, isobaric cooling from $T$ to $T_d$ would result in $w = w_s$.

The dewpoint depression, $\Delta T_d$, is defined as $T - T_d$. Note that $\Delta T_d = 0$ corresponds to an RH of 100%; larger values of $\Delta T_d$ correspond to relatively smaller RH.

In the absence of phase changes, mixing ratio is conserved, which implies that dewpoint remains on the same vapor line.
How does $w_s$ change with ascent?

For adiabatic ascent, pressure decreases, and by the Poisson equation, temperature also decreases. By the definition of $w_s$, a decrease in temperature should decrease its value, while a decrease in pressure should increase its value. Which change “wins out”?

Let’s evaluate the derivative $\frac{dw_s}{dp}$. After some algebra and using the IGL, 1st Law for an adiabatic process, and the Clausius-Clapeyron relationship, we find:

$$\frac{dw_s}{dp} \approx \frac{w_s}{p} \left( \frac{\varepsilon L}{c_p T} - 1 \right) > 0$$

Thus, during ascent, as $p$ decreases, saturation mixing ration also decreases.
Lifting condensation level (LCL)

Since mixing ratio is conserved for a dry adiabatic process, it is always possible to decrease \( p \) until \( w_s = w \), i.e., saturation is achieved. The pressure at which saturation is achieved is the **lifting condensation level** (LCL). The LCL defines the cloud base under forced ascent or free convection.

On a skew-T, the LCL of a parcel at temperature \( T \), dewpoint \( T_d \), and pressure \( p \) is determined by the intersection of the dry adiabat passing through \( T \) and vapor line passing through \( T_d \).