Homework #1
Due Thursday, September 17th, 2015

1. **Petty Problem 1.4** [10 pts]
2. **Petty Problem 1.5** [5 pts]
3. **Petty Problem 2.2** [5 pts]
4. **Petty Problem 2.3** [15 pts]
5. **Period of a Pendulum**
   In class, we invoked dimensional considerations to show that the period of a pendulum of length $L$ is proportional to $\sqrt{L/g}$. In this problem, you will show this more explicitly.
   **a** [5pts]. Assume that all of the pendulum’s mass $M$ is contained in the “bob” at the end of the pendulum rod. Write Newton’s 2nd Law for the force balance along the circular arc traced out by the pendulum bob as it swings back and forth. [Hint: You can relate the position of the pendulum bob along its arc to the angle $\theta$ formed by the pendulum rod and the vertical line when the pendulum bob is at the “base” of the arc. It may help to draw this.]
   **b** [5 pts]. Using the “small angle approximation” $\sin \theta(t) \approx \theta(t)$ and assuming that the pendulum bob is released from rest at an angle $\theta_0$, show that $\theta(t) = \theta_0 \cos (\omega t)$ solves the force balance equation in (a) for a particular value of $\omega$. What value is it?
   **c** [5 pts]. The period of the pendulum corresponds to the time it takes the pendulum to oscillate through one cycle, i.e., from its initial position and back. Use the result in b to find the period. You should find the period to be a numerical factor times the estimate of the period from dimensional considerations.

6. **Dimensional Analysis**
   The Stefan Boltzmann Law relates power emitted per unit area, $F$, by a so-called black body to its temperature $T$ as $F = \sigma T^4$ where $\sigma = \frac{2\pi^4 k_b^4}{15h^3c^3} = 5.67 \times 10^{-8}$ in SI units. Here $h$, $c$, and $k_b$ are fundamental constants.
   **a** [5 pts]. In terms of the dimensions of mass, $M$, length $L$, time $\tau$, and temperature $T$, what are the dimensions of $\sigma$? Please show how you arrived at this answer.
b [10 pts]. The Stefan Boltzmann Law can be derived from the so-called Planck function for a quantity called radiance, \( R(\lambda, T) = \frac{2hc^2}{\lambda^5[\text{e}^{\frac{hc}{\lambda k_b T}} - 1]} \), by the integral over \( R(\lambda, T) \) over wavelength \( \lambda \), \( F = \int_0^\infty R(\lambda, T) d\lambda \). By a suitable change of the variable of integration, show that the integral yields the 4\(^{th}\)-power dependence \( T \). [Hint: The argument of the exponential term is a dimensionless combination. With the change of variable, the integral one would need to evaluate can be rendered dimensionless by factoring out all of the dimensional quantities. It is not necessary to solve the integral!]