

- Convective quasi-equilibrium (QE) for the interaction of convection and large-scale dynamics and thermodynamics*

Goal: Understand the contemporary QE view of tropical convection

*Emanuel, K. A., J. D. Neelin, and C. S. Bretherton, 1994: On large-scale circulations in convecting atmospheres. *Q. J. Roy. Meteor. Soc.*, 120, 1111—1143.

Distinguishing two views on [or types of] convection

1. Large-scale (LS) processes generate CAPE which accumulates in the atmosphere until released through convective-scale processes.
2. LS generation of CAPE nearly balances consumption by convection, i.e., the generation and consumption processes are of comparable timescales.

Type 1: External/Triggered

- In this type of convection, the existence of CIN acts as a barrier to cumulus convection.
- Only sufficiently large perturbations in the vertical, or evolving low level conditions that relax the barrier, can generate convection.
- But unambiguously conditionally unstable profiles like the one schematically illustrated on the left have only been demonstrated over midlatitude continental areas.

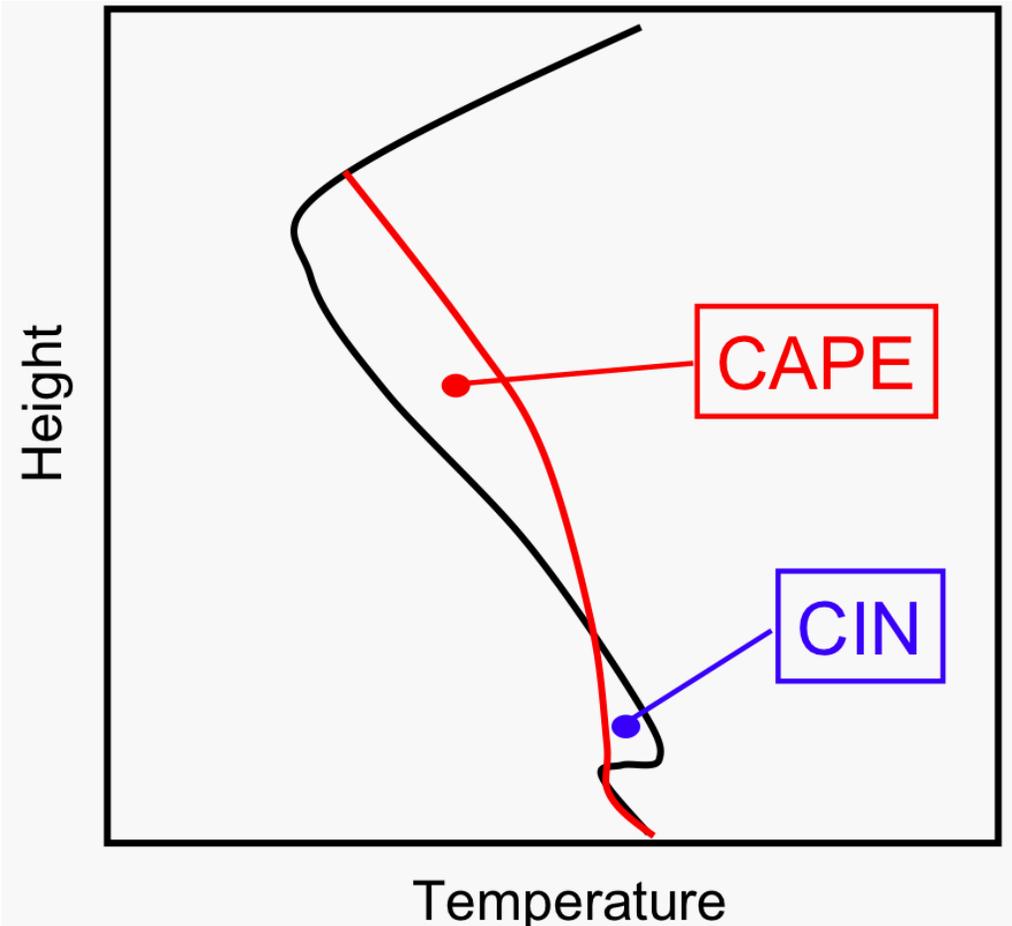


Figure by George Craig

Convection as a heat source for the circulation[?]

- In the external view, the energy released by convection “drives” the LS flow.
 - That is, the latent heat released typically exceeds the energy required to maintain the kinetic energy of LS motions against dissipation.
 - Thus, latent heating leads to KE production.
- However, this energy conversion requires a positive correlation between heating and temperature.
 - However, there is no reason *a priori* why this correlation must be positive.
 - In fact, while LS motions in the Tropics are associated with latent heat release, the cumulus convective heating and LS radiative and adiabatic cooling are nearly in balance: any residual (i.e., KE) is thus a small percentage of larger compensating terms.

Type 2: Internal/Statistical equilibrium

- The underlying notion of the internal, or statistical equilibrium view of convection, is that the timescale of the small-scale chaotic cumulus convective process is comparable to the LS processes with which it interacts.

“If we are to be able to predict the evolution of large-scale circulation systems that interact strongly with convection, and to do so on a time scale appropriate to the larger scale system (and not to the individual convective cells), then there had better be a strong relationship between the statistics of the convection and the properties of the large-scale system. If such a relationship does not exist, we are not entitled to believe that the larger scale system is predictable on timescales appreciably longer than those of convective cells.” - K. Emanuel

- The tropical free troposphere is often observed to be close to moist adiabatic. In the statistical equilibrium framework, convection is assumed to maintain the pervasive tropical moist adiabatic lapse rate and neutral stability of the tropical atmosphere.

Akio Arakawa*



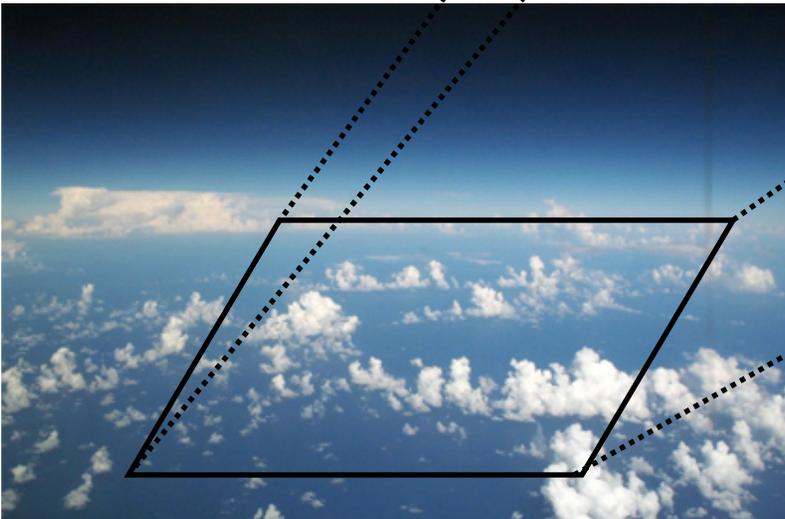
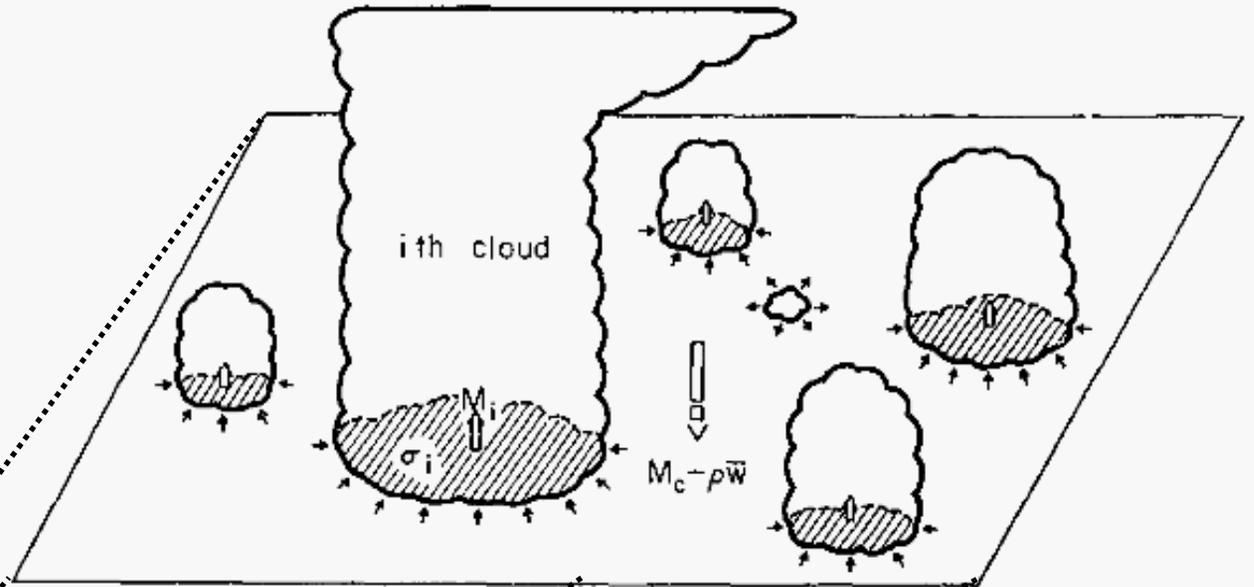
*“The 2010 Vilhelm Bjerknes Medal is awarded to Akio Arakawa in recognition of his pioneering and fundamental contributions to physically based discretisation techniques in atmosphere and ocean models and **to representations of convective clouds in atmosphere models**, and for his continuing work on bridging the gap between the resolved and unresolved scales in atmospheric general circulation modelling.”*

**Still going strong (age > 80 years) as a Professor Emeritus at UCLA*

Arakawa and Schubert (AS)

- Classic paper (published 1974), with the results used as the basis for many current generation convection schemes in models.
- Fastest energetic processes in convective clouds are:
 - 1) conversion of (available) PE to KE
 - 2) dissipation of KE
- Characteristic convective cloud timescales, e.g., the life cycle from development to decay, are of order hours, so the KE of a “cloud ensemble” in statistical equilibrium with the large-scale environment responds to changes in the large-scale conditions on similar timescales.

Cloud population



AS Overview: Subensembles and work functions

Consider a quantity λ that denotes a “subensemble” of convective clouds, characterized by a common attribute such as the altitude of the cloud tops. If M_λ and B_λ represent, respectively, the vertical mass flux and buoyancy of the λ subensemble, then the time rate of change of energy “consumed” by the λ subensemble is:

$$-\left(\frac{dA_\lambda}{dt}\right)_C = \int_{z_{b\lambda}}^{z_{t\lambda}} M_\lambda B_\lambda dz$$

Recall the definition of CAPE, which is of dimension $[B_\lambda dz] = \text{E/mass}$. Since $[M_\lambda]$ is mass/time, the integral is E/time.

Here, the limits of integration are the altitudes of cloud bottom and top, A_λ is the **cloud work function**, and the subscript C denotes a “cloud-scale” process.

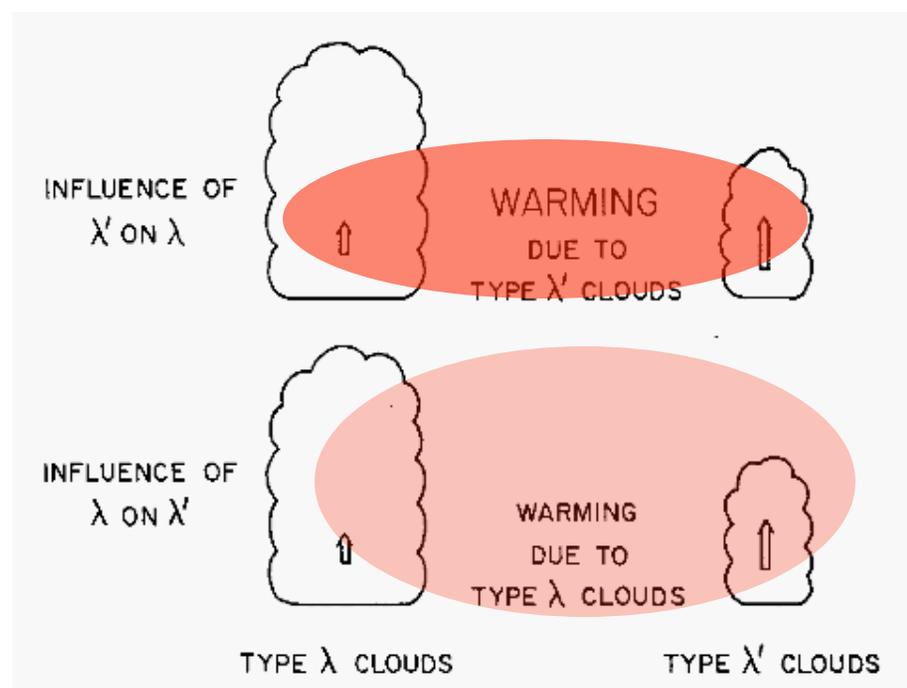
On the other hand, we can express the rate of energy production per unit time by the LS conditions for a given λ subensemble as:

$$\left(\frac{dA_\lambda}{dt}\right)_{LS} = F_\lambda - (1 - \delta_{\lambda\lambda'}) \int_{z_{b\lambda'}}^{z_{t\lambda'}} M_{\lambda'} J_{\lambda\lambda'} dz$$

Here, F_λ is the explicit LS rate of energy production and $J_{\lambda\lambda'}$ accounts for interactions between the λ and λ' subensembles.

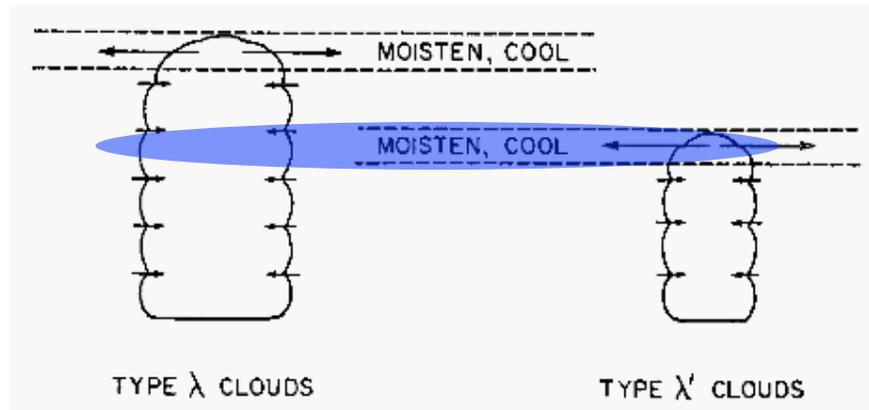
Interaction terms

Influence of vertical mass flux in λ' subensemble clouds on λ clouds (top) and λ clouds on λ' clouds (bottom)



Decrease cloud work function

Increase cloud work function



Influence of cloud-top detrainment of λ' subensemble clouds on λ clouds. Note for the interaction as shown, the effect of λ clouds on λ' clouds is zero, since the detrainment in the former occurs above the latter.

QE and AS

The quasi-equilibrium (QE) hypothesis means that the total time rate of change of the cloud work function is zero, implying the rate of LS energy production is equal to its consumption at the convective scale, i.e.,

$$\begin{aligned}\frac{dA_\lambda}{dt} &= \left(\frac{dA_\lambda}{dt}\right)_C + \left(\frac{dA_\lambda}{dt}\right)_{LS} = 0 \\ \Rightarrow \left(\frac{dA_\lambda}{dt}\right)_{LS} &= -\left(\frac{dA_\lambda}{dt}\right)_C\end{aligned}$$

That is:

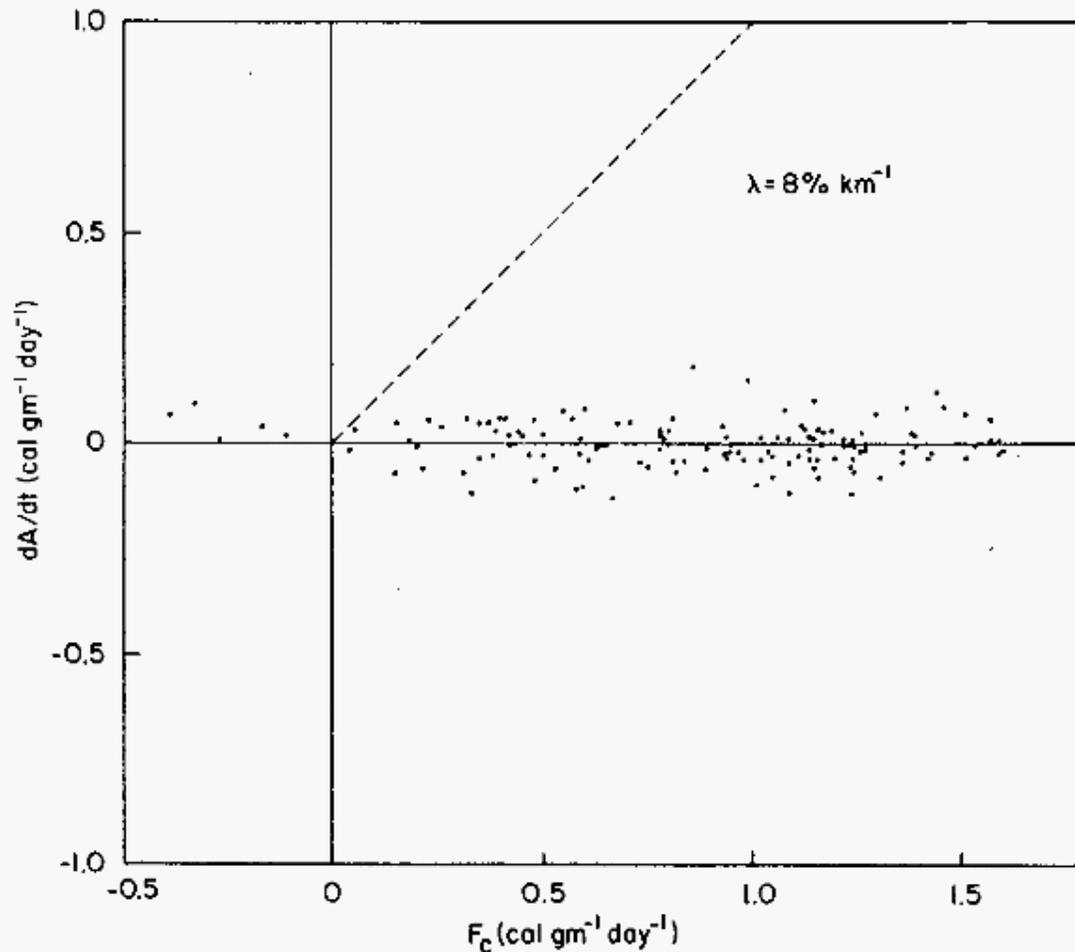
$$F_\lambda = \int_{z_{b\lambda}}^{z_{t\lambda}} M_{\lambda'} K_{\lambda\lambda'} dz$$

where the integration kernel $K_{\lambda\lambda'}$ is defined as:

$$K_{\lambda\lambda'} = (1 - \delta_{\lambda\lambda'}) J_{\lambda\lambda'} + \delta_{\lambda\lambda'} B_\lambda$$

Given F_λ and a cloud model that determines B_λ , M_λ can be determined: this is the basis of AS-type convection schemes used in models. However, this determination is nontrivial, as it depends on the features of the cloud models and how they treat cloud microphysics.

Observational confirmation of QE



- Figure from AS illustrating the time tendency of total cloud work function (y-axis) versus large-scale energy production (x-axis) from observations in the Marshall Islands (Yanai, 1973)
- For tropical conditions, net surface flux and column radiative cooling generate $\sim 4000 \text{ J kg}^{-1} \text{ day}^{-1}$, while CAPE values in the tropical atmosphere are typically below 1000 J kg^{-1} .

LS circulations in convecting atmospheres

- 1. Conditionally unstable conditions with appreciable CIN are unusual, especially in the Tropics.**
- 2. CAPE is approximately invariant.**
- 3. The invariance of CAPE has implications for the temperature of convecting atmospheres [next up]*
4. However, it's important to consider the small but nonzero response time of convection to large-scale forcing, which is important in tropical waves.

Relating ABL θ_e and cumulus convection

Consider the CAPE defined as: $CAPE = \int_{z_{LFC}}^{z_{EL}} g\rho^{-1}(\rho_{env} - \rho)dz$. Using the definition of specific volume and transforming the integral to one over pressure [through hydrostatic equilibrium] gives:

$$CAPE = \int_{p_{EL}}^{p_{LFC}} (\alpha - \alpha_{env}) dp$$

By “strict” QE, we have:

$$\frac{\partial CAPE}{\partial t} = \int_{p_{EL}}^{p_{LFC}} \left(\frac{\partial \alpha}{\partial t} - \frac{\partial \alpha_{env}}{\partial t} \right) dp \cong 0$$

Note there is no contribution to the time derivative from the limits of integration here.

In a prior lecture, we related constant-pressure perturbations in parcel specific volume to perturbations in entropy via Maxwell’s relations. Thus,

$$\left(\frac{\partial \alpha}{\partial t} \right)_p = \lim_{t \rightarrow 0} \frac{(\delta \alpha)_p}{\delta t} = \lim_{t \rightarrow 0} \left(\frac{\partial \alpha}{\partial s^*} \right)_p \frac{\delta s^*}{\delta t} = \left(\frac{\partial T}{\partial p} \right)_{s^*} \frac{\partial s^*}{\partial t} \quad s^* = c_p \ln \left(\frac{T}{T_0} \right) - R_d \ln \left(\frac{p}{p_0} \right) + \frac{L_c q_s}{T}$$

On the other hand, from the hydrostatic relationship, expressed in terms of geopotential:

$$\left(\frac{\partial \alpha_{env}}{\partial t} \right)_p = \frac{\partial}{\partial t} \left(- \frac{\partial \Phi}{\partial p} \right)$$

ABL θ_e and cumulus convection (cont'd)

Thus:

$$\frac{\partial \text{CAPE}}{\partial t} = \int_{p_{EL}}^{p_{LFC}} \left[\left(\frac{\partial T}{\partial p} \right)_{s^*} \frac{\partial s^*}{\partial t} + \frac{\partial}{\partial t} \left(\frac{\partial \Phi}{\partial p} \right) \right] dp \approx 0$$

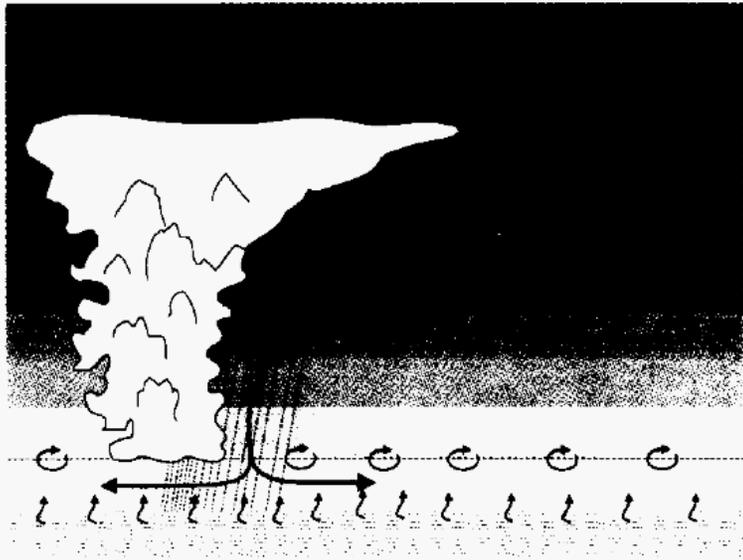
And:

$$\frac{\partial (c_p \ln \theta_e)_b}{\partial t} (T_{LFC} - T_{EL}) + \frac{\partial}{\partial t} (\Phi_{LFC} - \Phi_{EL}) \approx 0$$

We have assumed saturation entropy [or $c_p \ln \theta_e$] is equal to its ABL value and is further conserved following ascent within the convecting cloud.

The above relationship, namely that temporal fluctuations in the “thickness” of the (strict) QE convecting layer are proportional to ABL fluctuations in moist entropy/equivalent potential temperature, is a fundamental result. It implies that a positive perturbation in ABL θ_e produces a positive perturbation in thickness above the ABL (note: $\Phi_{EL} - \Phi_{LFC} > 0$). Thus, to a large extent in the Tropics, predicting the response of ABL θ_e to, say, large-scale disturbances will determine the impact of such disturbances on convection.

What determines equilibrium subcloud (ABL) θ_e ?



$$(\ln \theta_e)_{b eq} \cong \frac{1}{w_0 + w_e + M_d} \times \left[w_0 \ln(\theta_e)_s + w_e \ln(\theta_e)_e + M_d \ln(\theta_e)_d + \frac{z_b \dot{Q}_{rad}}{c_p T} \right]$$

Figure 4. Components of the entropy balance of the subcloud layer. The four major components are radiative cooling (not shown), surface sensible- and latent-heat fluxes, turbulent entrainment at the top of the subcloud layer, and unsaturated downdrafts driven by evaporation of precipitation.

Emanuel et al., 1994

Equilibrium subcloud θ_e increases with:

1. Increasing SST [i.e., higher θ_{es}]
2. Increasing surface windspeed [i.e., higher w_0]
3. Increasing above subcloud layer equivalent potential temperature [i.e., higher θ_{ee}]
4. Increasing downdraft equivalent potential temperature [i.e., higher θ_{ed}]