# TROPICAL METEOROLOGY HOMEWORK #2
# DUE THURSDAY 04/06/2015

***1. Equatorial wave group velocities [10 pts].***

The group velocity of a wave is the velocity of the overall shape of the wave's amplitude, or modulation, as it propagates through space. The group velocity represents the velocity at which energy is carried along a wave and is defined (in 1 spatial dimension) as $c_g = \frac{\partial \omega}{\partial k}$, where $\omega$ is the frequency of the wave and $k$ is the zonal wavenumber. Calculate the group velocities for equatorial Kelvin, equatorial Rossby, intertia-gravity, and mixed Rossby-gravity waves. Discuss how these compare to the phase velocities $c = \frac{\omega}{k}$ calculated in class?

***2. Geopotential perturbations in a moist adiabatic atmosphere [10 pts]***

a. Using the perturbation hydrostatic relationship in geopotential, i.e., $\frac{\partial \Phi'}{\partial p} = -\alpha'$,

and relating specific volume perturbations to a saturation entropy $s^{*\prime}$ that is constant in height, show that:

$$\Phi'(x, y, p, t) = \langle \Phi' \rangle + \left( \langle T \rangle - T(x, y, p, t) \right) s^{*\prime};$$

$$\langle \ldots \rangle = (p_s - p_t)^{-1} \int_{p_t}^{p_s} \ldots dp : (p_s, p_t) \equiv (surface, tropopause)$$

This relationship shows that geopotential perturbations in a moist adiabatic atmosphere consist of a barotropic (pressure-independent) part and a baroclinic part.

b. If the horizontal components of the windfield have an analogous structure to $\Phi'$

$$\mathbf{v}(x, y, p, t) = \langle \mathbf{v} \rangle + \left( \langle T \rangle - T(x, y, p, t) \right) \mathbf{v}^*$$

show that the pressure velocity at the tropopause is:

$$\omega(p_t) = (p_s - p_t) \nabla_h \cdot \langle \mathbf{v} \rangle$$

If the tropopause is assumed to be a rigid lid, this implies pressure velocity is zero there, so the divergence of the barotropic horizontal wind also vanishes.

***3. Changes in tropical precipitation and circulation under global warming [15 pts].***

A simple model has been proposed for understanding how precipitation and circulation in the tropics are expected to change under global warming. The starting point of this model assumes that tropical precipitation $P$ can be expressed as the product of the net upward mass flux $m$ in convecting regions and moisture $q$, $P = mq$.

a. Write a first order expansion for the change in precipitation, $\delta P$, in terms of $m$ and $q$.

b. The table below gives the change in saturation vapor pressure $e_s$ with temperature (as a result of the Clausis-Clapeyron equation). In this temperature range, what is the fractional change in saturation vapor pressure per K?

| Temperature (K) | Vapor Pressure of |
|---|---|

| | Water (mm Hg) |
|---|---|
| 288 | 12.81 |
| 289 | 13.63 |
| 290 | 14.53 |
| 291 | 15.48 |
| 292 | 16.48 |

c. The fractional change in precipitation with temperature as simulated by models under global warming is ~2%/K. Assuming constant relative humidity [$rh = \frac{q}{q_s} = \frac{e}{e_s}$] and the value computed in b as characteristic of the fractional change in saturation water vapor with temperature in these models, what is the implied fractional change in mass flux per K? What does this imply about the change in strength of tropical circulations under global warming?